

Towards natural inflation from weakly coupled heterotic string theory

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Abstract

We propose the natural inflation from the heterotic string theory on “Swiss-Cheese” Calabi-Yau manifold with multiple $U(1)$ magnetic fluxes. Such multiple $U(1)$ magnetic fluxes stabilize the same number of the linear combination of the universal axion and Kähler axions and one of the Kähler axions is identified as the inflaton. This axion decay constant can be determined by the size of one-loop corrections to the gauge kinetic function of the hidden gauge groups, which leads effectively to the trans-Planckian axion decay constant consistent with the WMAP, Planck and/or BICEP2 data. During the inflation, the real parts of the moduli are also stabilized by employing the nature of the “Swiss-Cheese” Calabi-Yau manifold.

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1 Introduction

The cosmological inflation is an accelerated expansion of the universe at the early universe which can solve the horizon problem and the flatness problem at the same time. Such expanding universe is realized by the vacuum energy density of the scalar field, so called inflaton, whose quantum fluctuations produce the origin of the density perturbation of the universe.

The current cosmological observations, especially, the Planck satellite report that the inflation scenario is well consistent with its data and the primordial density fluctuations are almost Gaussian and the spectrum index of the scalar density perturbation is nearly scale-invariant [1]. Recently, the BICEP2 collaboration reported that the primordial tensor modes can be measured as B-mode polarization of the Cosmic Microwave Background (CMB), which leads to the tensor-to-scalar ratio, $r = \mathcal{O}(0.1)$, although there is a tension between the Planck and BICEP2 collaborations. To achieve the tensor-to-scalar-ratio, $r = \mathcal{O}(0.1)$, we require that inflaton will roll slowly down to the minimum of its scalar potential from its Planckian field value which is problematic from the theoretical point of view, especially in the string theory which will be expected as the unified theory of gauge and gravitational interactions.

In the higher dimensional theories as well as the string theories, there are a lot of axions associated with the internal cycles of the internal manifolds such as the Calabi-Yau (CY) manifold which keeps the only $\mathcal{N} = 1$ supersymmetry (SUSY) in the four-dimensional (4D) spacetime. When we consider such axions as the candidate of inflaton, the natural inflation [2] is an attractive scenario as one of the large-field inflation, which is originally proposed by identifying the inflaton as the pseudo-Nambu Goldstone boson¹. However, the natural inflation compatible with the observed Planck [1] and/or BICEP2 [6] data requires the trans-Planckian axion decay constant, see Ref. [7] and references therein. So far, there are several approaches to realize the natural inflation with trans-Planckian axion decay constant in the framework of supergravity models or Type IIB superstring theory [8, 9, 10, 11, 12, 13, 14, 15, 16], although, in the string theory, the fundamental axion decay constants are typically in the range $10^{16} - 10^{17}\text{GeV}$ [17].

In this paper, we propose the single-field natural inflation in the framework of heterotic string theory on “Swiss-Cheese” CY manifold with multiple $U(1)$ fluxes induced from the anomalous $U(1)$ symmetries. By employing such multiple $U(1)$ fluxes, the linear combination of the universal axion and Kähler axions except for the inflaton are absorbed by the multiple $U(1)$ gauge bosons and they get the mass terms from these $U(1)$ fluxes. From the phenomenological point of view, $U(1)$ fluxes may be important tools to realize the 4D standard model gauge groups from the heterotic string theory [18, 19] as well as the Type IIB superstring theory [20]. During and after the inflation, the dilaton and real parts of Kähler moduli have to be stabilized and decoupled from the inflaton dynamics, otherwise the oscillations of these moduli would lead to the sizable isocurvature perturbations. In our model, the stabilization of the dilaton and real parts of Kähler moduli are realized by the non-perturbative corrections to the Kähler potential, superpotential and a nature of the structure of “Swiss-Cheese” CY manifold whose manifolds are also well studied as several topics about the particle phenomenology and cosmology based on the Type IIB string theory [21] or F-theory [22] and moduli stabilization based on the

¹The axion monodromy inflation is another interesting possibility. See e.g. [3, 4, 5].

heterotic string theory [23].

This paper is organized as follows. In Sec. 2, we review the heterotic string theory on CY manifold with multiple $U(1)$ magnetic fluxes. We propose two inflation models in Secs. 3.1 and 3.2. Both are consistent with the WMAP, Planck and/or BICEP2 data. Sec. 4 is devoted to the conclusion. In Appendix A, we show the mass matrices of fields for inflation model 1 in Sec. 3.1.

2 Heterotic string on CY manifolds with multiple $U(1)$ magnetic fluxes

We consider the $E_8 \times E_8$ or $SO(32)$ heterotic string theory on Calabi-Yau manifold with multiple $U(1)$ magnetic fluxes (in other words, multiple line bundles). The low-energy effective theory of the heterotic string is given by the following Lagrangian at the string frame,

$$S_{\text{bos}} = \frac{1}{2\kappa_{10}^2} \int_{M^{(10)}} e^{-2\phi_{10}} \left[R + 4d\phi_{10} \wedge *d\phi_{10} - \frac{1}{2} H \wedge *H \right] - \frac{1}{2g_{10}^2} \int_{M^{(10)}} e^{-2\phi_{10}} \text{tr}(F \wedge *F), \quad (1)$$

which is the bosonic part of the Lagrangian in the notation of [25] and ϕ_{10} is the dilaton and F is the field strength of $E_8 \times E_8$ or $SO(32)$ gauge groups and then “tr” denotes in the vector representation of these gauge groups. As will be mentioned later, the $U(1)$ magnetic fluxes are inserted in these gauge groups. H is the three-form field strength defined by

$$H = dB^{(2)} - \frac{\alpha'}{4}(w_{\text{YM}} - w_L), \quad (2)$$

where w_{YM} and w_L are the gauge and gravitational Chern-Simons three-form, respectively. The gravitational and Yang-Mills couplings are set by $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4$ and $g_{10}^2 = 2(2\pi)^7(\alpha')^3$.

Throughout this work, we focus on the $E_8^{\text{vis}} \times E_8^{\text{hid}}$ heterotic string with non-standard embedding, that is, the visible E_8^{vis} gauge group decomposes into the product group of G_{vis} and multiple $U(1)$ s where G_{vis} is the Grand Unified Group (GUT) or just the stand model (SM) gauge groups and we do not consider the charged scalar fields under the multiple $U(1)$ s². In addition to it, we assume that the hidden gauge groups are just non-abelian gauge groups, for simplicity.

After the dimensional reduction on the CY manifold with multiple $U(1)$ magnetic fluxes,

²An extension to the cases for $SO(32)$ heterotic string theory is straightforward.

we get the following 4D $U(1)$ invariant effective tree-level Kähler potential,

$$\begin{aligned} \mathcal{K} = & -M_{\text{Pl}}^2 \left[\ln \left(S + \bar{S} - \sum_m \frac{\mathcal{Q}_S^m}{16\pi^2} V_m \right) \right. \\ & \left. + \ln \left\{ \frac{d_{ijk}}{48} \left(T_i + \bar{T}_i - \sum_m \frac{\mathcal{Q}_{T_i}^m}{2\pi} V_m \right) \left(T_j + \bar{T}_j - \sum_m \frac{\mathcal{Q}_{T_j}^m}{2\pi} V_m \right) \left(T_k + \bar{T}_k - \sum_m \frac{\mathcal{Q}_{T_k}^m}{2\pi} V_m \right) \right\} \right], \end{aligned} \quad (3)$$

where $M_{\text{Pl}}^2 = \frac{e^{-2\phi_{10}} \mathcal{V}}{\kappa_{10}^2}$, m labels the number of anomalous $U(1)$ vector multiplets V_m , d_{ijk} are the intersection numbers of the Calabi-Yau manifold. S and T_i for $i = 1, 2, \dots, h^{1,1}$ are the superfield descriptions of the dilaton and the Kähler moduli, respectively,

$$\begin{aligned} S &= \frac{1}{4\pi} \left[\frac{e^{-2\phi_{10}} \mathcal{V}}{l_s^6} + i b_S^{(0)} \right], \\ T_i &= t_i + i b_{T_i}^{(0)}, \end{aligned} \quad (4)$$

where $\mathcal{V} = \frac{1}{6} \int_{\text{CY}} J \wedge J \wedge J$ with $J = l_s^2 \sum_i t_i w_i$ is the volume of the CY manifold, J is the Kähler form expanded by the base of two-form w_i , $i = 1, \dots, h^{1,1}$ and $l_s = 2\pi\sqrt{\alpha'}$ is the string length. The imaginary parts of S and T_i , $b_S^{(0)}$ and $b_{T_i}^{(0)}$ are the universal and Kähler axions given by the dimensional reduction of the Kalb-Ramond two-form $B^{(2)}$ and six-form $B^{(6)}$ as

$$\begin{aligned} B^{(2)} &= b_S^{(2)} + l_s^2 \sum_{k=1}^{h_{11}} b_{T_k}^{(0)} w_k, \\ B^{(6)} &= l_s^6 b_S^{(0)} \text{vol}_6 + l_s^4 \sum_{k=1}^{h_{11}} b_{T_k}^{(2)} \hat{w}_k, \end{aligned} \quad (5)$$

where $b_S^{(2)}$ and $b_{T_i}^{(2)}$ are the 4D tensor fields, vol_6 is the normalized volume form, $\int_{\text{CY}} \text{vol}_6 = 1$, and \hat{w}_i are the Hodge dual four-form of the two-form w_i . The two-form $B^{(2)}$ and six-form $B^{(6)}$ are related by the Hodge duality, $*_{10} dB^{(2)} = e^{2\phi_{10}} dB^{(6)}$ and $*_4 db_I^{(2)} = e^{2\phi_{10}} db_I^{(0)}$, $I = S, T_1, \dots, T_{h^{1,1}}$.

The $U(1)$ charges of the dilaton and Kähler moduli for the $U(1)^m$ symmetries, \mathcal{Q}_I^m , $I = S, T_1, \dots, T_{h^{1,1}}$ are defined via the following couplings of the $U(1)$ gauge bosons A_m ,

$$S \supset \sum_m \frac{\mathcal{Q}_S^m}{4l_s^2} \int_{R^{1,3}} b_S^{(2)} \wedge F_m + \sum_{i,m} \frac{\mathcal{Q}_{T_i}^m}{2l_s^2} \int_{R^{1,3}} b_{T_i}^{(2)} \wedge F_m, \quad (6)$$

where

$$\mathcal{Q}_S^m \equiv \text{tr}(T^m T^m) \int_{\text{CY}} \frac{\text{tr} \bar{F}_m}{2\pi} \wedge \frac{1}{16\pi^2} \left(\text{tr} \bar{F}^2 - \frac{1}{2} \text{tr} \bar{R}^2 \right), \quad \mathcal{Q}_{T_i}^m \equiv \text{tr}(T^m T^m) \int_{T_i} \frac{\text{tr} \bar{F}_m}{2\pi}, \quad (7)$$

T^m are the $U(1)^m$ generators embedded in the visible E_8 gauge group, \bar{F}_m and \bar{F} are the internal field strengths of the $U(1)^m$ symmetry and E_8^{vis} symmetry. Such couplings are obtained from

the dimensional reduction of the 10D kinetic terms of H given by Eq. (1) and the one-loop Green-Schwarz (GS) counter term [24] which is determined by the S-dual of the type I theory as shown in Appendix of Ref. [19],

$$S_{\text{GS}} = \frac{1}{24(2\pi)^5\alpha'} \int B^{(2)} \wedge X_8, \quad (8)$$

where the eight-form X_8 reads,

$$X_8 = \frac{1}{24} \text{Tr} F^4 - \frac{1}{7200} (\text{Tr} F^2)^2 - \frac{1}{240} (\text{Tr} F^2) (\text{tr} R^2) + \frac{1}{8} \text{tr} R^4 + \frac{1}{32} (\text{tr} R^2)^2. \quad (9)$$

The mass terms of the $U(1)$ gauge bosons are derived by expanding the Kähler potential to second order on the vector multiplets,

$$S_{\text{mass}} = - \sum_{m,n} \frac{M_{\text{Pl}}^2}{4} \left(\frac{K_{S\bar{S}} \mathcal{Q}_S^m \mathcal{Q}_{\bar{S}}^n}{(16\pi^2)^2} + \sum_{i,j} \frac{K_{T_i\bar{T}_j} \mathcal{Q}_{T_i}^m \mathcal{Q}_{\bar{T}_j}^n}{(2\pi)^2} \right) \int_{R^{1,3}} A_m \wedge *_4 A_n, \quad (10)$$

which is typically of order the string scale $M_s^2 = 1/l_s^2$ with $l_s = 2\pi\sqrt{\alpha'}$, see Refs. [26] for $E_8 \times E_8$ and [28] for $SO(32)$ heterotic string theories and references therein.

From the $U(1)$ invariant Kähler potential given by Eq. (3), $U(1)$ magnetic fluxes generate the moduli-dependent Fayet-Iliopoulos terms [29],

$$\xi_m = \left. \frac{\partial \mathcal{K}}{\partial V_m} \right|_{V_m=0} = - \frac{\mathcal{Q}_S^m}{16\pi^2} K_S - \sum_{i=1}^{h^{1,1}} \frac{\mathcal{Q}_{T_i}^m}{2\pi} K_{T_i}, \quad (11)$$

where $K_I = \partial \mathcal{K} / \partial Z^I$ for $Z^I = S, T_1, \dots, T_{h^{1,1}}$. Finally, we comment on the gauge kinetic function of the non-abelian gauge groups obtained from the decomposition of the $E_8^{(\text{vis})} \times E_8^{(\text{hid})}$ heterotic string theory. They receive the one-loop corrections originating from the one-loop GS term as shown in Eq. (8),

$$\begin{aligned} f_{\text{vis}} &= S + \beta_i T_i, \\ f_{\text{hid}} &= S - \beta_i T_i, \end{aligned} \quad (12)$$

where

$$\beta_i \equiv \frac{1}{8\pi} \int_{\text{CY}} \frac{1}{16\pi^2} \left(\text{tr} \bar{F}^2 - \frac{1}{2} \text{tr} \bar{R}^2 \right) \wedge \hat{w}_i. \quad (13)$$

Both gauge kinetic functions in the visible and hidden sector are correlated by the tadpole cancellation condition of the $E_8^{(\text{vis})} \times E_8^{(\text{hid})}$ heterotic string theory. For the $SO(32)$ heterotic string theory, the non-abelian gauge groups included in $SO(32)$ have the nonuniversal gauge kinetic functions depend on the decomposition of $SO(32)$.

3 Natural inflation from heterotic string theory

In this section, we propose two natural inflation scenarios in the framework of the weakly coupled heterotic string theory on “Swiss-Cheese” Calabi-Yau manifold with multiple $U(1)$ fluxes induced from the anomalous $U(1)$ symmetries. As pointed out in the introduction, both natural inflation scenarios are the single-filed inflation models whose inflaton are identified as the single Kähler axion with trans-Planckian axion decay constant. The trans-Planckian axion decay constant is originating from the one-loop corrections to the gauge kinetic function of the hidden gauge groups to achieve the successful natural inflation which is different from the natural inflation scenarios by employing two axions with sub-Planckian axion decay constants [30].

On the other hand, the other Kähler axions are absorbed by the multiple $U(1)$ gauge bosons and become massive. The real part of dilaton is stabilized at the finite value by the contributions from the non-perturbative effect to the dilaton Kähler potential and gaugino condensation term as shown in Secs. 3.1 and 3.2, respectively. One of the real parts of Kähler moduli is stabilized by the world sheet instanton effect which leads to the stabilization of other real parts of Kähler moduli.

3.1 Model 1 (Single gaugino condensation)

In this section, we will show the inflaton potential along the following three steps. First, the universal axion and Kähler axions except for the inflaton are absorbed by the multiple $U(1)$ gauge bosons at the string scale as shown in Eq. (10). Next, the dilaton and all real parts of Kähler moduli are stabilized at the SUSY breaking minimum by the inclusion of non-perturbative corrections to the dilaton Kähler potential and superpotential which is the world sheet instanton effect. Finally, below the SUSY breaking scale, we get the effective scalar potential for the light Kähler axion which is identified as the inflaton later.

3.1.1 Setup

We consider the following Kähler potential with five Kähler moduli and four anomalous $U(1)$ symmetries,

$$\begin{aligned} \mathcal{K} = & K(S + \bar{S}, V^1, V^2, V^3) \\ & - \ln \left\{ k_1 (T_1 + \bar{T}_1)^3 - k_2 \left(T_2 + \bar{T}_2 - \sum_{n=1}^3 q_{T_2}^n V^n \right)^3 - k_3 \left(T_3 + \bar{T}_3 - \sum_{n=1}^3 q_{T_3}^n V^n \right)^3 \right. \\ & \left. - k_4 (T_4 + \bar{T}_4 - q_{T_4}^4 V^4)^3 - k_5 (T_5 + \bar{T}_5 - q_{T_5}^4 V^4)^3 \right\}, \end{aligned} \quad (14)$$

in the unit $M_{\text{Pl}} = 1$ with $M_{\text{Pl}} = 2.4 \times 10^{18} \text{GeV}$, where we choose $h^{1,1} = 5$, $q_{T_i}^m = \mathcal{Q}_{T_i}^m / 2\pi$, $i = 1, 2, 3, 4, 5$, $m = 1, 2, 3, 4$ and k_i , $i = 1, 2, 3, 4, 5$ are the positive constants determined by the intersection numbers $d_{t_1 t_1 t_1}$, $d_{t_2 t_2 t_2}$, $d_{t_3 t_3 t_3}$, $d_{t_4 t_4 t_4}$, $d_{t_5 t_5 t_5}$. (The reason why we choose five Kähler moduli and four anomalous $U(1)$ s are shown later.)

The Kähler potential of dilaton consists of the tree-level and the non-perturbative part,

$$\begin{aligned} K^0 &= -\ln \left(S + \bar{S} - \sum_{n=1}^3 q_S^n V^n \right), \\ K^{\text{np}} &= d g^{-p} e^{-b/g}, \end{aligned} \quad (15)$$

where b , p , and d are the real constants, $q_s^n = \mathcal{Q}_S^n / 16\pi^2$, $n = 1, 2, 3$ and $g = (\text{Re } S - \sum_{i \neq 1} \beta_i \text{Re } T_i)^{-1/2}$ is the gauge coupling in the hidden sector as shown in the superpotential (12). K^{np} denotes the non-perturbative correction to the Kähler potential [31, 32, 33]. There are known ansatz to write the dilaton Kähler potential as

$$K = K^0 + K^{\text{np}} \quad \text{or} \quad K = \ln \left(e^{K^0} + e^{K^{\text{np}}} \right), \quad (16)$$

etc.³. Anyway, we assume that the dilaton is stabilized at the finite value due to such corrections to the Kähler potential as discussed in [34]. Note that our following moduli stabilization as well as the inflation mechanism do not depend on the detailed structure of the non-perturbative Kähler potential, K^{np} .

In addition to the Kähler potential, we consider the following $U(1)^m$, $m = 1, 2, 3, 4$, invariant superpotential,

$$W = W_0 + A e^{-\frac{8\pi^2}{a}(S - \beta_2 T_2 - \beta_3 T_3 - \beta_4 T_4 - \beta_5 T_5)} + B e^{-\mu_1 T_1}, \quad (17)$$

where W_0 is the Neveu-Schwarz (NS) three-form flux induced constant term which stabilizes the $h^{1,2}$ complex structure moduli of the CY manifold, the second term of the right handed side (r.h.s.) shows the hidden sector gaugino condensation which receive the one-loop corrections originating from the one-loop Green-Schwarz counter term given by Eq. (13). The third term of the (r.h.s.) shows the world-sheet instanton effects on the two cycle T_1 .

As the first step to obtain the inflaton potential, we comment on the anomalous $U(1)^m$ vector multiplets V^m , $m = 1, 2, 3, 4$. Such $U(1)^m$ vector multiplets are massive due to the $U(1)^m$ magnetic fluxes as shown in Eq. (10) and then $U(1)^m$ gauge bosons absorb the linear combination of imaginary component of the dilaton and the Kähler moduli. $U(1)^1$, $U(1)^2$ and $U(1)^3$ gauge bosons absorb the following linear combination of the moduli,

$$\begin{aligned} X^1 &= \frac{1}{N^1} \left(\frac{\text{Im } S}{q_S^1 \sqrt{K_{S\bar{S}}}} + \frac{\text{Im } T_2}{q_{T_2}^1 \sqrt{K_{T_2\bar{T}_2}}} + \frac{\text{Im } T_3}{q_{T_3}^1 \sqrt{K_{T_3\bar{T}_3}}} \right), \\ X^2 &= \frac{1}{N^2} \left(\frac{\text{Im } S}{q_S^2 \sqrt{K_{S\bar{S}}}} + \frac{\text{Im } T_2}{q_{T_2}^2 \sqrt{K_{T_2\bar{T}_2}}} + \frac{\text{Im } T_3}{q_{T_3}^2 \sqrt{K_{T_3\bar{T}_3}}} \right), \\ X^3 &= \frac{1}{N^3} \left(\frac{\text{Im } S}{q_S^3 \sqrt{K_{S\bar{S}}}} + \frac{\text{Im } T_2}{q_{T_2}^3 \sqrt{K_{T_2\bar{T}_2}}} + \frac{\text{Im } T_3}{q_{T_3}^3 \sqrt{K_{T_3\bar{T}_3}}} \right), \end{aligned} \quad (18)$$

³In [32], the non-perturbative Kähler potential of dilaton is discussed in the effective field theory approaches.

where $N^i = \sqrt{(1/q_S^n \sqrt{K_{S\bar{S}}})^2 + (1/q_{T_2}^n \sqrt{K_{T_2\bar{T}_2}})^2 + (1/q_{T_3}^n \sqrt{K_{T_3\bar{T}_3}})^2}$, $n = 1, 2, 3$, respectively. Here the dilaton and Kähler moduli are canonically normalized and their Kähler metric are summarized in Appendix A.

Thus, the imaginary component of S , T_2 and T_3 are absorbed by the $U(1)^1$, $U(1)^2$, $U(1)^3$ gauge bosons and their mass-squared matrices are given by

$$M_{m,n}^2 \simeq \frac{M_{\text{Pl}}^2}{4\sqrt{\langle \text{Re } f_{m,m} \rangle} \sqrt{\langle \text{Re } f_{n,n} \rangle}} \left(K_{S\bar{S}} q_S^m q_S^n + \sum_{i,j} K_{T_i\bar{T}_j} q_{T_i}^m q_{T_j}^n \right) \quad (19)$$

for $m, n = 1, 2, 3$, where the $U(1)$ gauge bosons are canonically normalized. The gauge kinetic functions of $U(1)$ s, $f_{m,n}$ are given by $f_{m,n} = \text{tr}(T^m T^n) S \delta_{m,n} + \mathcal{O}(\beta T)$. These $U(1)^1$, $U(1)^2$ charges of the moduli S , T_2 and T_3 are related by the $U(1)^1$, $U(1)^2$ and $U(1)^3$ gauge invariance of the superpotential (17),

$$q_S^1 = q_{T_2}^1 \beta_2 + q_{T_3}^1 \beta_3, \quad q_S^2 = q_{T_2}^2 \beta_2 + q_{T_3}^2 \beta_3, \quad q_S^3 = q_{T_2}^3 \beta_2 + q_{T_3}^3 \beta_3, \quad (20)$$

Under the $U(1)$ gauge invariance condition (20), the full-rank mass matrices (19) are realized if the number of $U(1)$ s are bigger than three. Thus we can stabilize the Kähler axion and universal axion. The universal axion cannot be identified as the candidate of inflaton, because its decay constant is much less than the Planck scale as shown in the superpotential (17).

The other $U(1)^4$ gauge boson absorbs the following combination of the moduli,

$$X^4 = \frac{1}{N^4} \left(\frac{\text{Im } T_4}{q_{T_4}^4 \sqrt{K_{T_4\bar{T}_4}}} + \frac{\text{Im } T_5}{q_{T_5}^4 \sqrt{K_{T_5\bar{T}_5}}} \right), \quad (21)$$

where $N^4 = \sqrt{(1/q_{T_4}^4 \sqrt{K_{T_4\bar{T}_4}})^2 + (1/q_{T_5}^4 \sqrt{K_{T_5\bar{T}_5}})^2}$, and the orthogonal direction of X_4 (which is identified as the inflaton later),

$$Y^4 = \frac{1}{N^4} \left(-\frac{\text{Im } T_4}{q_{T_5}^4 \sqrt{K_{T_5\bar{T}_5}}} + \frac{\text{Im } T_5}{q_{T_4}^4 \sqrt{K_{T_4\bar{T}_4}}} \right), \quad (22)$$

cannot be absorbed by the anomalous $U(1)$ gauge bosons and the mass of Y^4 is obtained from the gaugino condensation term in Eq. (17). In summary, the four imaginary parts of the moduli X^m , $m = 1, 2, 3, 4$ are absorbed by four anomalous $U(1)$ vector multiplets after following the above procedure. As the second step to obtain the inflaton potential, let us discuss the F-term potential derived from the Kähler potential (14) and the superpotential (17).

First, we redefine the linear combination of the dilaton and the Kähler moduli as,

$$\Phi = S - \beta_2 T_2 - \beta_3 T_3 - \beta_4 T_4 - \beta_5 T_5, \quad (23)$$

and then the Kähler potential and superpotential given by Eqs. (14) and (17) are rewritten by

$$\begin{aligned}
\mathcal{K} = & K(\Phi + \bar{\Phi}, T_2 + \bar{T}_2, T_3 + \bar{T}_3, T_4 + \bar{T}_4, T_5 + \bar{T}_5, V^1, V^2, V^3) \\
& - \ln \left\{ k_1(T_1 + \bar{T}_1)^3 - k_2 \left(T_2 + \bar{T}_2 - \sum_{n=1}^3 q_{T_2}^n V^n \right)^3 - k_3 \left(T_3 + \bar{T}_3 - \sum_{n=1}^3 q_{T_3}^n V^n \right)^3 \right. \\
& \left. - k_4 (T_4 + \bar{T}_4 - q_{T_4}^4 V^4)^3 - k_5 (T_5 + \bar{T}_5 - q_{T_5}^4 V^4)^3 \right\}, \\
W = & W_0 + A e^{-\frac{8\pi^2}{a} \Phi} + B e^{-\mu_1 T_1},
\end{aligned} \tag{24}$$

and we assume that the gaugino condensation term in Eq. (24) is much smaller than the other terms in Eq. (24) at least at the minimum, that is, $W_0, B e^{-\mu_1 T_1} \gg A e^{-\frac{8\pi^2}{a} \Phi}$. Therefore, at the moment, we ignore the contribution of the gaugino condensation term as we will mention later.

Second, we stabilize the moduli $T_1, \text{Re } T_2, \text{Re } T_3, \text{Re } T_4, \text{Re } T_5$ and $\text{Re } \Phi$ by imposing the supersymmetric conditions,

$$\begin{aligned}
D_{T_1} W &= 0, \\
D_{T_2} W &= K_{T_2} W = 0, \quad D_{T_3} W = K_{T_3} W = 0, \quad D_{T_4} W = K_{T_4} W = 0, \quad D_{T_5} W = K_{T_5} W = 0 \\
D_{\Phi} W &= K_{\Phi} W = 0,
\end{aligned} \tag{25}$$

where $D_I W = W_I + K_I W$ and $W_I = \partial W / \partial Z_I$ with $Z_I = T_1, T_2, T_3, T_4, T_5, \Phi$. Then the D-term potential induced from the Kähler potential (14) are automatically vanished under the above supersymmetric conditions, $K_{T_2} = K_{T_3} = K_{T_4} = K_{T_5} = K_{\Phi} = 0$. $\text{Re } \Phi$ is stabilized by the contribution from the non-perturbative correction to the dilaton,

$$K_{\Phi} = 0. \tag{26}$$

In the same way for $\text{Re } \Phi$, the real parts of moduli T_j , $j = 2, 3, 4, 5$, are stabilized by the following conditions,

$$K_{T_j} \simeq \frac{3k_j(T_j + \bar{T}_j)^2}{k_1(T_1 + \bar{T}_1)^3} + \frac{\partial K^0}{\partial T_j} \simeq \frac{3k_j(T_j + \bar{T}_j)^2}{k_1(T_1 + \bar{T}_1)^3} - \frac{\beta_j}{\Phi + \bar{\Phi}} + \mathcal{O} \left(\beta_j \frac{\sum_{k=2}^5 \beta_k \text{Re } T_k}{\text{Re } \Phi} \right) = 0, \tag{27}$$

where the dilaton Kähler potential is approximated as its tree-level part K^0 in Eq. (16). In the limit of $\text{Re } T_1 > \text{Re } T_j$, $j = 2, 3, 4, 5$, the above equations are rewritten by

$$\text{Re } S \simeq \text{Re } \Phi \simeq \frac{k_1(\text{Re } T_1)^3}{3k_j \text{Re } T_j^2} \beta_j \gg \beta_j \text{Re } T_j, \tag{28}$$

for $j = 2, 3, 4, 5$. Thus the tree-level part of the gauge kinetic function is always bigger than the one-loop corrections of one under the condition that $\text{Re } T_1 > \text{Re } T_j$ ($j \neq 1$) as shown in Eq. (12), that is, the perturbative expansion is valid. This property is an important feature of the ‘‘Swiss-Cheese’’ Calabi-Yau manifold.

From the scalar potential given by using the formula of 4D $N = 1$ supergravity,

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \quad (29)$$

we get the supersymmetric AdS minimum at the minimum given by Eq. (25),

$$\langle V \rangle = -3e^K |W|^2. \quad (30)$$

There are several approaches to uplift such AdS vacuum by the F-terms with dynamical SUSY breaking sector [35, 36, 37, 38] or D-terms with anti-heterotic five branes [39], etc.. Here we assume that the SUSY is broken by the dynamical SUSY breaking sector whose Kähler potential and superpotential are given by

$$\begin{aligned} \Delta K &= |X|^2 - \frac{|X|^4}{\Lambda^2}, \\ \Delta W &= \mu X, \end{aligned} \quad (31)$$

where X is gauge singlet chiral superfield under the non-abelian groups in the visible sector G_{vis} and anomalous $U(1)^m$ symmetries, $m = 1, 2, 3, 4$, Λ is the dynamical SUSY breaking scale and we omit the moduli dependence of X , because they do not affect the following moduli stabilization. Then the Minkowski minimum is realized by choosing the parameter μ as

$$\langle V \rangle + \Delta V \simeq e^{\langle K \rangle} \left(-3|\langle W \rangle|^2 + K^{X\bar{X}} |\mu|^2 \right) = 0 \Leftrightarrow |\mu|^2 = 3|\langle W \rangle|^2. \quad (32)$$

Finally, we consider the contribution of the omitted term $Ae^{-\frac{8\pi^2}{\alpha}\Phi}$ in the superpotential (24) which is ignored on the previous analysis. Since we assume that such omitted term is much smaller than the other terms in the superpotential (24) at the minimum, the moduli $\text{Re } \Phi$, T_1 , $\text{Re } T_2$, $\text{Re } T_3$, $\text{Re } T_4$ and $\text{Re } T_5$ are stabilized at the minimum close to the values given by Eq. (25) and are decoupled from the inflaton dynamics if their masses are heavier than the inflation scale. The mass scales of these moduli are determined by the constant term, the world-sheet instanton effect of the superpotential (24) and the D-term contribution (14) which is heavier than the inflation scale as shown later. (The mass matrices of them are summarized in the Appendix A.). As mentioned before, the imaginary parts of the moduli except for the inflaton Y^4 are absorbed by the four $U(1)$ gauge bosons whose mass scale is of order the string scale M_s .

3.1.2 Inflaton potential and its dynamics

Now we are ready to write down the inflation potential. As discussed in the previous section 3.1.1, after integrating out these heavy moduli and substituting the field values given by Eq. (25), we get the effective scalar potential for the light moduli Y^4 which is the linear combination of $\text{Im } T_4$ and $\text{Im } T_5$ given by Eq. (22),

$$V_{\text{eff}} \simeq \Lambda^4 (1 - \cos(\beta \hat{Y}^4)), \quad (33)$$

in the limit of $Ae^{-\frac{8\pi^2}{a}\langle\text{Re}\Phi\rangle} \ll W_0, Be^{-\mu_1\langle T_1\rangle}$, where the energy scale of the scalar potential Λ^4 and the axion decay constant β are defined as

$$\begin{aligned}\Lambda^4 &\equiv 6e^K e^{-\frac{8\pi^2}{a}\text{Re}\Phi} A(W_0 + Be^{-\mu_1 T_1}), \\ \beta &\equiv \frac{8\pi^2}{aN^4\hat{N}^4} \left(\frac{\beta_5}{q_{T_4}^4 \sqrt{K_{T_4\bar{T}_4}}} - \frac{\beta_4}{q_{T_5}^4 \sqrt{K_{T_5\bar{T}_5}}} \right),\end{aligned}\tag{34}$$

and

$$\hat{Y}^4 \simeq \frac{1}{N^4} \sqrt{2 \left(\frac{K_{T_4\bar{T}_4}}{(q_{T_5}^4)^2 K_{T_5\bar{T}_5}} + \frac{K_{T_5\bar{T}_5}}{(q_{T_4}^4)^2 K_{T_4\bar{T}_4}} \right)} Y^4 \equiv \hat{N}^4 Y^4\tag{35}$$

is the canonically normalized axion field. Here we employed the following redefinitions of the moduli,

$$\begin{aligned}\text{Im } T_4 &= \frac{1}{N^4} \left(\frac{X^4}{q_{T_4}^4 \sqrt{K_{T_4\bar{T}_4}}} - \frac{Y^4}{q_{T_5}^4 \sqrt{K_{T_5\bar{T}_5}}} \right), \\ \text{Im } T_5 &= \frac{1}{N^4} \left(\frac{X^4}{q_{T_5}^4 \sqrt{K_{T_5\bar{T}_5}}} - \frac{Y^4}{q_{T_4}^4 \sqrt{K_{T_4\bar{T}_4}}} \right),\end{aligned}\tag{36}$$

and $U(1)^4$ gauge invariance of the superpotential (24),

$$q_{T_4}^4 \beta_4 + q_{T_5}^4 \beta_5 = 0.\tag{37}$$

When we identify the axion \hat{Y}^4 as the inflaton, the effective scalar potential (33) is considered as the inflation potential for the single-field \hat{Y}^4 , since the mass of the other moduli are much heavier than the inflaton. Thus we can realize the scalar potential of the type of natural inflation. The power spectrum of the scalar density perturbation is explained by choosing the parameter, $\Lambda^4 \sim \mathcal{O}(10^{-9})$ in the M_{Pl} unit, and the spectral index of the scalar density perturbation and the tensor-to-scalar ratio is also consistent with the cosmological observations reported by WMAP, Planck and/or BICEP2 collaborations. This is because we can realize the trans-Planckian axion decay constant β originating from the one-loop corrections to the gauge kinetic function as shown in Eq. (34).

Next, we estimate the cosmological observables constrained by the observations. We choose the dilaton Kähler potential as the type of $K^0 + K^{\text{np}}$ ⁴ and the following input parameters in the Kähler potential given by Eqs. (14) and (15) as

$$\begin{aligned}k_1 &= k_2 = k_3 = k_4 = k_5 = \frac{1}{8}, \\ d &= 7, \quad b = 1, \quad p = 2, \\ \beta_2 &\simeq \beta_3 \simeq \beta_4 \simeq \beta_5 \simeq 0.01,\end{aligned}\tag{38}$$

⁴The stabilization of moduli are discussed in Appendix A.

and in the superpotential given by Eqs. (17) and (31) as,

$$A = \frac{1}{300}, \quad a = 30, \quad B = -\frac{1}{2}, \quad \mu_1 = 2\pi, \quad W_0 = 6 \times 10^{-4}, \quad \mu \simeq 1 \times 10^{-3}, \quad (39)$$

in the unit $M_{\text{Pl}} = 1$ and the $U(1)$ charges of the moduli are of $\mathcal{O}(1)$. From these input parameters, we get the field values of the moduli at the minimum,

$$T_1 \simeq 1.3, \quad T_2 \simeq T_3 \simeq T_4 \simeq T_5 \simeq 0.06, \quad S \simeq \Phi \simeq 2, \quad (40)$$

which yield the gauge coupling unification of the grand unified theory (GUT) at the Kaluza-Klein (KK) scale,

$$M_{KK} \simeq \frac{M_s}{\mathcal{V}^{1/6}} \simeq 1.2 \times 10^{17} \text{ GeV}, \quad (41)$$

with

$$M_s = \frac{M_{\text{Pl}}}{\sqrt{4\pi\alpha^{-1}}} \simeq 1.4 \times 10^{17} \text{ GeV}, \quad (42)$$

where $\alpha^{-1} \simeq 24$ is the gauge coupling of visible gauge group G_{vis} at the string scale.

By employing the input parameters given by Eqs. (38) and (39), the energy scale of the scalar potential,

$$\Lambda^4 \simeq 3.22 \times 10^{-9}, \quad (43)$$

and the axion decay constant,

$$\beta^{-1} \simeq 7.8, \quad (44)$$

in the unit $M_{\text{Pl}} = 1$, are obtained which leads to the desired trans-Planckian axion decay constant.

To estimate the cosmological observables, we define the slow-roll parameters,

$$\begin{aligned} \epsilon &\equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{\partial_{\hat{Y}^4} V_{\text{eff}}}{V_{\text{eff}}} \right)^2, \\ \eta &\equiv M_{\text{Pl}}^2 \frac{\partial_{\hat{Y}^4}^2 V_{\text{eff}}}{V_{\text{eff}}}, \\ \xi^2 &\equiv M_{\text{Pl}}^4 \frac{\partial_{\hat{Y}^4} V_{\text{eff}} \partial_{\hat{Y}^4}^3 V_{\text{eff}}}{V_{\text{eff}}^2}, \end{aligned} \quad (45)$$

and then the e-folding number from the time t_* to the inflation end t_{end} is estimated as

$$N_e = - \int_{t_{\text{end}}}^{t_*} dt H(t) \simeq \frac{1}{M_{\text{Pl}}} \int_{\hat{Y}_*^4}^{\hat{Y}_{\text{end}}^4} \frac{d\hat{Y}^4}{\sqrt{2\epsilon}}, \quad (46)$$

where the Hubble parameter $H(t)$ is defined as $H(t) = \frac{\dot{a}(t)}{a(t)}$, $a(t)$ is the scale factor of the 4D spacetime. \hat{Y}_*^4 and \hat{Y}_{end}^4 are the field values of the inflaton \hat{Y}^4 at the time t_* and t_{end} , respectively.⁵ The observables such as the power spectrum of the scalar density perturbation P_ζ , the spectral index of it n_s , the tensor-to-scalar ratio r are written in terms of the slow-roll parameters as

$$\begin{aligned} P_\zeta &= \frac{1}{24\pi^2} \frac{V_{\text{eff}}}{\epsilon M_{\text{Pl}}^4}, \\ n_s &= 1 - 6\epsilon + 2\eta, \\ r &= 16\epsilon. \end{aligned} \tag{47}$$

At the field value $\hat{Y}_*^4 \simeq 13M_{\text{Pl}}$, we find the numerical values of observables and the e-folding number as

$$P_\zeta \simeq 2.2 \times 10^{-9}, \quad n_s \simeq 0.956, \quad r \simeq 0.11, \quad N_e \simeq 48, \tag{48}$$

which are consistent with the WMAP, Planck data [1],

$$P_\zeta = 2.196_{-0.060}^{+0.051} \times 10^{-9}, \quad n_s = 0.9583 \pm 0.0080, \tag{49}$$

at the pivot scale $k_* = 0.05 \text{Mpc}^{-1}$ and BICEP2 data [6],

$$r = 0.16_{-0.05}^{+0.06}, \tag{50}$$

after considering the foreground dust. Now we choose the hidden gauge group as E_8 which leads to the dual Coxeter number $a = 30$ and $\beta_3 \simeq \beta_4 \simeq \beta_5 \simeq 0.01$.

Note that we can realize smaller tensor-to-scalar ratio which is more consistent with the WMAP and Planck data, since the size of the axion decay constant β depends on the dual Coxeter number of the hidden gauge group a in Eq. (34) and the size of one-loop correction to the gauge kinetic function of the hidden gauge group in Eq. (13).

3.2 Model 2 (Double gaugino condensations)

In this section, we propose the natural inflation based on the other type of the Kähler potential and superpotential. The main difference between the model 1 in the previous Sec. 3.1 and the model 2 in this section is the stabilization mechanism of dilaton. In the model 1, the dilaton is stabilized at the finite value by the non-perturbative corrections to its Kähler potential given by Eq. (16). However, in the model 2, the dilaton is stabilized by using one of the gaugino condensation terms which will be mentioned later. In the same way as the model 1, the trans-Planckian axion decay constant is realized from the one-loop correction to the gauge kinetic function of the hidden gauge group.

⁵The end of inflation is estimated when the slow-roll condition is violated as $\max\{|\epsilon|, |\eta|\} = 1$.

3.2.1 Setup

We consider the CY manifold expressed by the following Kähler potential with one $U(1)$ anomalous symmetry,

$$\mathcal{K} = -\ln(S + \bar{S}) - \ln \left(k_b (T_b + \bar{T}_b)^3 - k_s \left(T_s + \bar{T}_s - \frac{\mathcal{Q}_s}{2\pi} V_s \right)^3 - k'_s \left(T'_s + \bar{T}'_s - \frac{\mathcal{Q}'_s}{2\pi} V_s \right)^3 \right), \quad (51)$$

in the unit $M_{\text{Pl}} = 1$, where $h^{1,1} = 3$, k_b , k_s , k'_s are positive constants determined by the triple intersection number $d_{t_b t_b t_b}$, $d_{t_s t_s t_s}$, $d_{t'_s t'_s t'_s}$ and V_s is an anomalous $U(1)_s$ vector multiplet under which only two moduli T_s and T'_s have $U(1)_s$ charge. $U(1)_s$ vector multiplet absorbs the linear combination of Kähler axions, while the other massless axion is identified as the inflaton. We further assume that the dilaton Kähler potential is approximated by its tree-level Kähler potential.

Next, we consider the following $U(1)_s$ invariant superpotential,

$$W = w_0 + A_2 e^{-\frac{8\pi^2}{a_2}(S - \beta_s^{(1)} T_s - \beta_s'^{(1)} T'_s)} + B_2 e^{-\frac{8\pi^2}{b_2}(S - \beta_s^{(2)} T_s - \beta_s'^{(2)} T'_s)} + C_2 e^{-\mu_b T_b}, \quad (52)$$

where w_0 is the NS flux induced constant term which stabilizes the $h^{1,2}$ complex structure moduli of the CY manifold, the second and third term of the right handed side (r.h.s.) show the gaugino condensations on two hidden sectors, the fourth term of the (r.h.s.) shows the world-sheet instanton effect on the two-cycle T_b . Let us discuss the moduli stabilization and the inflaton potential.

First, $U(1)_s$ vector multiplet becomes massive whose mass scale is of order the string scale due to the $U(1)_s$ magnetic fluxes as shown in Eq. (10), and then $U(1)_s$ gauge boson absorbs the linear combination of $\text{Im } T_s$ and $\text{Im } T'_s$ as,

$$X_s = \frac{1}{N_s} \left(\frac{\text{Im } T_s}{q_s \sqrt{K_{T_s \bar{T}_s}}} + \frac{\text{Im } T'_s}{q'_s \sqrt{K_{T'_s \bar{T}'_s}}} \right), \quad (53)$$

where $N_s = \sqrt{(1/q_s \sqrt{K_{T_s \bar{T}_s}})^2 + (1/q'_s \sqrt{K_{T'_s \bar{T}'_s}})^2}$ with $q_s = \mathcal{Q}_s/2\pi$ and $q'_s = \mathcal{Q}'_s/2\pi$ and two Kähler moduli are canonically normalized under the condition that their Kähler mixing are neglected, because their stabilization is also the same as the previous model 1 in Sec. 3.1. Its orthogonal direction (which is identified as the inflaton later),

$$Y_s = \frac{1}{N_s} \left(-\frac{\text{Im } T_s}{q'_s \sqrt{K_{T'_s \bar{T}'_s}}} + \frac{\text{Im } T'_s}{q_s \sqrt{K_{T_s \bar{T}_s}}} \right), \quad (54)$$

remains massless. The $U(1)_s$ charges of the moduli are related as

$$\begin{aligned} q_s \beta_s^{(1)} + q'_s \beta_s'^{(1)} &= 0, \\ q_s \beta_s^{(2)} + q'_s \beta_s'^{(2)} &= 0, \end{aligned} \quad (55)$$

due to the $U(1)_s$ gauge invariance of the superpotential (52).

Second, we redefine a linear combination of dilaton and Kähler moduli as,

$$\Phi = S - \beta_s^{(1)} T_s - \beta_s'^{(1)} T'_s, \quad (56)$$

and then the Kähler potential and superpotential are rewritten by

$$\begin{aligned} \mathcal{K} = & -\ln(\Phi + \bar{\Phi} + \beta_s^{(1)}(T_s + \bar{T}_s) + \beta_s'^{(1)}(T'_s + \bar{T}'_s)) \\ & -\ln\left(k_b(T_b + \bar{T}_b)^3 - k_s\left(T_s + \bar{T}_s - \frac{\mathcal{Q}_s}{2\pi}V_s\right)^3 - k'_s\left(T'_s + \bar{T}'_s - \frac{\mathcal{Q}'_s}{2\pi}V_s\right)^3\right), \\ W = & w_0 + A_2 e^{-\frac{8\pi^2}{a_2}\Phi} + B_2 e^{-\frac{8\pi^2}{b_2}(\Phi + (\beta_s^{(1)} - \beta_s^{(2)})T_s + (\beta_s'^{(1)} - \beta_s'^{(2)})T'_s)} + C_2 e^{-\mu_b T_b}, \end{aligned} \quad (57)$$

and we assume that first and second and fourth terms of the (r.h.s.) in Eq. (57) are much larger than the third term of the (r.h.s.) in Eq. (57) at least at the minimum, that is, $w_0, A_2 e^{-\frac{8\pi^2}{a_2}\Phi}, C_2 e^{-\mu_b T_b} \gg B_2 e^{-\frac{8\pi^2}{b_2}(\Phi + (\beta_s^{(1)} - \beta_s^{(2)})T_s + (\beta_s'^{(1)} - \beta_s'^{(2)})T'_s)}$. Such hierarchies between two gaugino condensation terms are realized by the differences between the rank of the two hidden gauge groups whose gauginos condensate. Therefore, at the moment, we ignore the term $B_2 e^{-\frac{8\pi^2}{b_2}(\Phi + (\beta_s^{(1)} - \beta_s^{(2)})T_s + (\beta_s'^{(1)} - \beta_s'^{(2)})T'_s)}$ in the superpotential (57).

Third, we stabilize the moduli Φ , T_b , $\text{Re}T_s$ and $\text{Re}T'_s$ by imposing the supersymmetric conditions,

$$\begin{aligned} D_\Phi W &= 0, \\ D_{T_b} W &= 0, \\ K_{T_s} &= K_{T'_s} = 0, \end{aligned} \quad (58)$$

which leads to the vanishing D -terms induced from the Kähler potential (57). From the scalar potential in the framework of 4D $N = 1$ supergravity given by Eq. (29), we get the supersymmetric AdS minimum at the minimum given by Eq. (58),

$$\langle V \rangle = -3e^K |W|^2. \quad (59)$$

In the same way as the previous model 1 in the Sec. 3.1, here we assume that the dynamical SUSY breaking sector uplift this AdS minimum. Their Kähler potential and superpotential are given by

$$\begin{aligned} \Delta K &= |X|^2 - \frac{|X|^4}{\Lambda^2}, \\ \Delta W &= \mu X, \end{aligned} \quad (60)$$

where X is the gauge singlet chiral superfield under the non-abelian gauge groups G_{vis} and anomalous $U(1)_s$ symmetry, Λ is the dynamical SUSY breaking scale and we omit the moduli dependence of X , because they do not affect the following moduli stabilization. The Minkowski minimum is realized by choosing the parameter μ as

$$\langle V \rangle + \Delta V \simeq e^K \left(-3|W|^2 + K^{X\bar{X}} |\mu|^2 \right) = 0, \quad \Leftrightarrow |\mu|^2 = 3|\langle W \rangle|^2. \quad (61)$$

Finally, we consider the term $B_2 e^{-\frac{8\pi^2}{b_2}(\Phi + (\beta_s^{(1)} - \beta_s^{(2)})T_s + (\beta_s'^{(1)} - \beta_s'^{(2)})T_s')}$ in the superpotential (57). Since we assume that such term is much smaller than the other terms in the superpotential (57), the moduli Φ , T_b , $\text{Re}T_s$, $\text{Re}T_s'$ are stabilized at the values close to the minimum given by Eq. (58) and they become massive due to the constant term of the superpotential for Φ , $\text{Re}T_s$ and $\text{Re}T_s'$ and the world-sheet instanton effect for T_b . As $\text{Re}T_s$ and $\text{Re}T_s'$, they also obtain the D-term contributions from the Kähler potential (57).

3.2.2 Inflaton potential

Let us discuss the inflaton potential. After integrating out these heavy moduli and substituting the field values given by Eq. (58), we get the effective scalar potential for the light modulus Y_s which is the linear combination of $\text{Im}T_s$ and $\text{Im}T_s'$,

$$V_{\text{eff}} \simeq \Lambda_s^4 (1 - \cos(\beta_s \hat{Y}_s)), \quad (62)$$

in the limit of $B_2 e^{-\frac{8\pi^2}{b_2} \langle \text{Re}\Phi \rangle} \ll w_0$, $A_2 e^{-\frac{8\pi^2}{a_2} \langle \text{Re}\Phi \rangle}$, $C_2 e^{-\mu_b \langle T_b \rangle}$, where

$$\Lambda_s^4 \equiv 6e^K e^{-\frac{8\pi^2}{b_2} \text{Re}\Phi} B_2 (w_0 + A_2 e^{-\frac{8\pi^2}{a_2} \Phi} + C_2 e^{-\mu_b T_b}), \quad (63)$$

and the axion decay constant β_s is defined by

$$\beta_s \equiv \frac{8\pi^2}{b_2 N_s \hat{N}_s} \left(-\frac{\beta_s^{(1)} - \beta_s^{(2)}}{q'_s \sqrt{K_{T_s' \bar{T}_s'}}} + \frac{\beta_s'^{(1)} - \beta_s'^{(2)}}{q_s \sqrt{K_{T_s \bar{T}_s}}} \right). \quad (64)$$

\hat{Y}_s is the canonically normalized axion field,

$$\hat{Y}_s \simeq \frac{1}{N_s} \sqrt{2 \left(\frac{K_{T_s \bar{T}_s}}{(q'_s)^2 K_{T_s' \bar{T}_s'}} + \frac{K_{T_s' \bar{T}_s'}}{(q_s)^2 K_{T_s \bar{T}_s}} \right)} Y_s \equiv \hat{N}_s Y_s. \quad (65)$$

Here we employed the following redefinitions of the moduli,

$$\begin{aligned} \text{Im} T_s &= \frac{1}{N_s} \left(\frac{X_s}{q_s \sqrt{K_{T_s \bar{T}_s}}} - \frac{Y_s}{q'_s \sqrt{K_{T_s' \bar{T}_s'}}} \right), \\ \text{Im} T_s' &= \frac{1}{N_s} \left(\frac{X_s}{q'_s \sqrt{K_{T_s' \bar{T}_s'}}} + \frac{Y_s}{q_s \sqrt{K_{T_s \bar{T}_s}}} \right), \end{aligned} \quad (66)$$

and $U(1)_s$ gauge invariance of the superpotential (52),

$$\begin{aligned} q_s \beta_s^{(1)} + q'_s \beta_s'^{(1)} &= 0, \\ q_s \beta_s^{(2)} + q'_s \beta_s'^{(2)} &= 0. \end{aligned} \quad (67)$$

Thus when we consider the axion \hat{Y}_s as the inflaton, the effective scalar potential is the type of natural inflation. The power spectrum of the scalar density perturbation is explained by choosing the parameter,

$$\Lambda_s^4 \sim \mathcal{O}(10^{-9}) \quad (68)$$

in the M_{Pl} unit, and the spectral index of the scalar density perturbation and the tensor-to-scalar ratio are also consistent with the cosmological observations reported by WMAP, Planck and BICEP2 collaborations. This is because we can realize the trans-Planckian axion decay constant β_s originating from the one-loop corrections to the gauge kinetic function.

However, $E_8 \times E_8$ or $SO(32)$ heterotic string theories have the rank 16 gauge groups which have to incorporate the rank 4 SM gauge groups. Then the energy scales which two gaugino condense are constrained since the total rank of their gauge groups are taken up to 12 included in $E_8 \times E_8$ or $SO(32)$. Thus we would need tuning some parameters to realize the correct inflation scale.

4 Conclusion

In this paper, we proposed two natural inflation scenarios based on the weakly coupled $E_8 \times E_8$ or $SO(32)$ heterotic string theory on the “Swiss-cheese” Calabi-Yau manifold with multiple $U(1)$ magnetic fluxes. The natural inflation is consistent with the WMAP, Planck and/or BICEP2 data, only if the size of axion decay constant becomes the trans-Planckian. However, such trans-Planckian axion decay constant is problematic from the theoretical point of view, especially on the supergravity models or the string theory. So far, there are known scenarios to get the trans-Planckian axion decay constant from the sub-Planckian axion decay constants [30].

We identified the inflaton as one of the linear combination of Kähler axions associated with the two-cycles of the Calabi-Yau manifold. When the gauginos of the hidden gauge group are condensed, the gaugino condensation terms are generated on the superpotential in the framework of 4D $\mathcal{N} = 1$ supergravity. In this case, we can realize the trans-Planckian axion decay constant originating from the one-loop corrections to the gauge kinetic function of the hidden gauge group derived from one-loop Green-Schwarz term [24] which is the feature of the weakly coupled heterotic string theory. On the other hand, in type II superstring theory such as the intersecting D-models or magnetized D-branes, the gauge kinetic function has the $\mathcal{O}(1)$ moduli mixing induced from the winding number of D-brane, magnetic fluxes or instanton effects.

To realize the single-field inflaton potential, we have to stabilize the dilaton and the other Kähler moduli. At the same time, their masses should be heavier than the inflation scale, otherwise these moduli would be oscillated during and after the inflation which may lead to the sizable isocurvature perturbations and cosmological moduli problem. Therefore, we considered two stabilization scenarios categorized as the model 1 and 2 based on the $E_8 \times E_8$ or $SO(32)$ heterotic string theory with multiple $U(1)$ magnetic fluxes.

In the case of model 1 discussed in the Sec. 3.1, the dilaton is stabilized at the finite value by the contributions from its non-perturbative corrections to the Kähler potential. The volume

moduli is also stabilized by the world-sheet instanton effect which leads to the stabilization of the other real parts of Kähler moduli by using the nature of “Swiss-cheese” Calabi-Yau manifold. By employing the multiple $U(1)$ magnetic fluxes, the imaginary parts of the moduli except for the inflaton are absorbed by the corresponding anomalous $U(1)$ gauge bosons and then they become massive which is of order the string scale. Thus we can realize the single-field axion potential with trans-Planckian axion decay constant determined by the one-loop corrections to the gauge kinetic function of the hidden gauge group.

The essential difference between the model 1 and 2 is the stabilization mechanism of dilaton. In model 2 discussed in the Sec. 3.2, the dilaton is stabilized by one of the gaugino condensation terms and we get the effective scalar potential for a linear combination of the Kähler axions. These two proposed inflation scenarios are consistent with the WMAP, Planck and/or BICEP2 data, although we need to tune the parameters in the model 2.

We can also realize smaller tensor-to-scalar ratio which is more consistent with the WMAP and Planck data, since the size of axion decay constant depends on the dual Coxeter number and the size of one-loop correction to the gauge kinetic function of the hidden gauge group in the heterotic string theory.

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A Mass matrices in model 1

In this appendix, we show the mass-squared matrices of the scalar potential given by the Kähler potential (24) and superpotential (24) in the model 1. As we have seen in Sec. 3.1, the moduli are stabilized at the value given by the supersymmetric conditions, $K_I = 0$ with $I = \Phi, T^2, T^3, T^4$ and they will become massive due to the constant superpotential and D-term contributions as shown later. For completeness, we assume the ansatz of the dilaton Kähler potential such as $K = K^0 + K^{\text{np}}$ in Eq. (16).

First, we canonically normalize the moduli to estimate their masses. In the case of Kähler potential (24) whose dilaton Kähler potential is replaced with $K = K^0 + K^{\text{np}}$ in Eq. (16), the non-vanishing Kähler mixing of the dilaton and Kähler moduli are expanded in the limit of

$\text{Re } S \gg \beta_j \text{Re } T_j$ and $T_1 \gg T_j$, $j = 2, 3, 4, 5$,

$$\begin{aligned}
K_{\Phi\bar{\Phi}} &\simeq -\frac{b}{16} \frac{2}{(\Phi + \bar{\Phi})^{3/2}} K^{\text{np}} + \frac{1}{2} \left(p - b \left(\frac{\Phi + \bar{\Phi}}{2} \right)^{1/2} \right) \frac{1}{(\Phi + \bar{\Phi})^2}, \\
K_{\Phi\bar{T}_j} &\simeq \frac{\beta_j}{(\Phi + \bar{\Phi})^2}, \\
K_{T_1\bar{T}_1} &\simeq \frac{3}{(T_1 + \bar{T}_1)^2}, \\
K_{T_1\bar{T}_j} &\simeq \frac{9k_j(T_j + \bar{T}_j)^2}{k_1(T_1 + \bar{T}_1)^4}, \\
K_{T_j\bar{T}_j} &\simeq \frac{6k_j(T_j + \bar{T}_j)}{k_1(T_1 + \bar{T}_1)^3}, \\
K_{T_i\bar{T}_j} &\simeq \frac{9k_i k_j (T_i + \bar{T}_i)^2 (T_j + \bar{T}_j)^2}{k_1^2 (T_1 + \bar{T}_1)^6},
\end{aligned} \tag{69}$$

with $i \neq j$, $i, j = 2, 3, 4, 5$. Here we use the following stabilization conditions of the moduli,

$$\begin{aligned}
K_{\Phi} &\simeq -\frac{1}{\Phi + \bar{\Phi}} + \frac{1}{2(\Phi + \bar{\Phi})} \left(p - b \left(\frac{\Phi + \bar{\Phi}}{2} \right)^{1/2} \right) K^{\text{np}} = 0, \\
K_{T_j} &\simeq \frac{3k_j(T_j + \bar{T}_j)^2}{k_1(T_1 + \bar{T}_1)^3} - \frac{\beta_j}{\Phi + \bar{\Phi}} = 0,
\end{aligned} \tag{70}$$

for $j = 2, 3, 4, 5$. As discussed in Sec. 3.1, the perturbative expansion is ensured under the above stabilization conditions, that is, $S \gg \beta_j T_j$ for $j = 2, 3, 4, 5$. Since the off-diagonal elements are suppressed by the smallness of β_j and the value of moduli T_j , $j = 2, 3, 4, 5$ at the minimum given by Eq. (70), the moduli Kähler metric are approximated by their diagonal form,

$$K_{I\bar{J}} \simeq K_{I\bar{J}} \delta_{I\bar{J}}, \tag{71}$$

with $I, J = \Phi, T_1, T_j$ for $j = 2, 3, 4, 5$.

Second, we show the mass matrices given by the D-term potential which is obtained from the Kähler potential (24) whose dilaton Kähler potential is replaced with $K = K^0 + K^{\text{np}}$ in Eq. (16),

$$\begin{aligned}
V_D &= \frac{1}{2f_{U(1)^1}} (q_S^1 K_S + q_{T_2}^1 K_{T_2} + q_{T_3}^1 K_{T_3})^2 + \frac{1}{2f_{U(1)^2}} (q_S^2 K_S + q_{T_2}^2 K_{T_3} + q_{T_3}^2 K_{T_3})^2 \\
&\quad + \frac{1}{2f_{U(1)^3}} (q_S^3 K_S + q_{T_2}^3 K_{T_2} + q_{T_3}^3 K_{T_3})^2 + \frac{1}{2f_{U(1)^4}} (q_{T_4}^4 K_{T_4} + q_{T_5}^4 K_{T_5})^2,
\end{aligned} \tag{72}$$

where the gauge kinetic functions of $U(1)^m$, $m = 1, 2, 3, 4$ are approximated as $f_{U(1)^m} \simeq \text{tr}(T^m T^m) S$. At the SUSY minimum where $K_I = 0$ with $I = \Phi, T_2, T_3, T_4, T_5$ and $D_{T_1} W = 0$, the second derivatives of the above D-term potential can be expanded in the small parameter β_j , $j = 2, 3, 4, 5$ as

$$(V_D)_{I\bar{J}} = (V_D)_{I\bar{J}}^0 + (V_D)_{I\bar{J}}^1 + \dots \tag{73}$$

where

$$\begin{aligned}
(V_D)_{\Phi\bar{\Phi}}^0 &= \sum_{n=1}^3 \frac{1}{2f_{U(1)^n}} (q_S^n K_{\Phi\bar{\Phi}} + q_{T_2}^n K_{T_2\bar{\Phi}} + q_{T_3}^n K_{T_3\bar{\Phi}})^2, \\
(V_D)_{\Phi\bar{T}_2}^0 &= \sum_{n=1}^3 \frac{1}{2f_{U(1)^n}} (q_S^n K_{\Phi\bar{\Phi}} + q_{T_2}^n K_{T_2\bar{\Phi}}) q_{T_2}^n K_{T_2\bar{T}_2}, \\
(V_D)_{\Phi\bar{T}_3}^0 &= \sum_{n=1}^3 \frac{1}{2f_{U(1)^n}} (q_S^n K_{\Phi\bar{\Phi}} + q_{T_3}^n K_{T_3\bar{\Phi}}) q_{T_3}^n K_{T_3\bar{T}_3}, \\
(V_D)_{T_2\bar{T}_2}^0 &= \sum_{n=1}^3 \frac{1}{2f_{U(1)^n}} (q_{T_2}^n)^2 (K_{T_2\bar{T}_2})^2, \\
(V_D)_{T_2\bar{T}_3}^0 &= \sum_{n=1}^3 \frac{1}{2f_{U(1)^n}} q_{T_2}^n q_{T_3}^n K_{T_2\bar{T}_2} K_{T_3\bar{T}_3}, \\
(V_D)_{T_3\bar{T}_3}^0 &= \sum_{n=1}^3 \frac{1}{2f_{U(1)^n}} (q_{T_3}^n)^2 (K_{T_3\bar{T}_3})^2, \\
(V_D)_{T_4\bar{T}_4}^0 &= \frac{1}{2f_{U(1)^4}} (q_{T_4}^4 K_{T_4\bar{T}_4} + q_{T_5}^4 K_{T_4\bar{T}_5})^2, \\
(V_D)_{T_4\bar{T}_5}^0 &= \frac{1}{2f_{U(1)^4}} (q_{T_4}^4 K_{T_4\bar{T}_4} + q_{T_5}^4 K_{T_4\bar{T}_5}) (q_{T_4}^4 K_{T_4\bar{T}_5} + q_{T_5}^4 K_{T_5\bar{T}_5}), \\
(V_D)_{T_5\bar{T}_5}^0 &= \frac{1}{2f_{U(1)^4}} (q_{T_4}^4 K_{T_4\bar{T}_5} + q_{T_5}^4 K_{T_5\bar{T}_5})^2,
\end{aligned} \tag{74}$$

and the other elements of the mass matrices are vanishing. As can be seen in Eq. (69), $(V_D)_{I\bar{J}}^0$ are of order β_j^2 , $j = 2, 3, 4, 5$. On the other hand, we find that $(V_D)_{I\bar{J}}^1$ and $(V_D)_{I\bar{J}}^2$ are of order β_j^3 and β_j^4 , respectively and they are smaller than the mass terms obtained from the F-term contributions in the case of input parameters given by Eqs. (38) and (39).

The mass matrices given by the F-term potential which is obtained from the Kähler potential and superpotential (24) are shown as

$$\begin{aligned}
(V_F)_{\Phi\bar{\Phi}} &\simeq e^K K^{\Phi\bar{\Phi}} |K_{\Phi\bar{\Phi}} W|^2, \\
(V_F)_{T_1\bar{T}_1} &\simeq e^K K^{T_1\bar{T}_1} |W_{T_1}|^2, \\
(V_F)_{T_2\bar{T}_2} &\simeq e^K K^{T_2\bar{T}_2} |K_{T_1\bar{T}_1} W|^2, \\
(V_F)_{T_3\bar{T}_3} &\simeq e^K K^{T_3\bar{T}_3} |K_{T_3\bar{T}_3} W|^2, \\
(V_F)_{T_4\bar{T}_4} &\simeq e^K K^{T_4\bar{T}_4} |K_{T_4\bar{T}_4} W|^2, \\
(V_F)_{T_5\bar{T}_5} &\simeq e^K K^{T_5\bar{T}_5} |K_{T_5\bar{T}_5} W|^2,
\end{aligned} \tag{75}$$

and other elements of the mass matrices are vanishing at the minimum given by Eqs. (25). Here the Kähler metric is approximated as the diagonal form and we neglect the gaugino condensation term in Eq. (24).

Finally we show the total mass matrices given by the D-term and F-term potential approximated as

$$(V)_{I\bar{J}} \simeq (V_D)_{I\bar{J}}^0 + (V_F)_{I\bar{J}}, \quad (76)$$

and we find that this mass-squared matrices are full-rank and the eigenvalues of them are positive in the choice of the input parameters. Their mass scales of the moduli are determined by the string scale and SUSY breaking scale $m_{3/2} = e^{\langle K \rangle / 2} \langle W \rangle \simeq 5 \times 10^{14} \text{ GeV}$ from the D-term and F-term contributions, respectively.

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